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**FACULTY OF ENGINEERING AND TECHNOLOGY**

**DEPARTMENT OF COMPUTER ENGINEERING**

**COURSE TITLE: FEEDBACK SYSTEMS LABORATORY**

**COURSE CODE: EEF 460**

LAB 4

# QUINUEL TABOT NDIP-AGBOR

# FE21A300

**Course Supervisor:**

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**Question 1 :**

**a)** q(s) = s5 + s4 + 3s3 +4s2 +s + 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Row | s^5 | s^4 | s^3 | s^2 | s |
| 1 | 1 | 1 | 3 | 4 | 1 |
| 2 | 1 | (3 - 4) = -1 | 4 + 3 | (4^1 – 3^1) = 1 |  |
| 3 | 1 | -1 | 7 |  |  |

**Analysis Of Sign Changes:**

There is one sign change in the first column (from 1 to -1).

**Interpretation:** Since there's one sign change, the system has **one root** in the right-half plane (positive real part). Therefore, the system is **unstable**

**b)** q(s) = s5 + s4 + 4s3 +4s2 + 2s + 1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Row | s^5 | s^4 | s^3 | s^2 | s |
| 1 | 1 | 1 | 4 | 4 | 2 |
| 2 | 1 | (4 - 4) = 0 | (4^2 – 4^1) = 4 | (4^1 – 2^2) = 0 |  |
| 3 | 1 | 0 | 4 |  |  |

**Analysis Of Sign Changes:**

There are zero sign changes in the first column.

**Interpretation:** Since there are no sign changes, the system has **all roots** in the left-half plane (negative real parts). Therefore, the system is **stable**.

**c)** q(s) = s8 + 3s7 + 5s6 + 6s5 + s4 + 4s3 +4s2 + 2s + 1

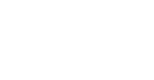
|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 5 | 1 | 4 |
| 3 | 6 | 4 | 0 |
| 0 | 4 | 1 | 0 |
| 6 | 2 | 0 | 0 |
| 4 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |

**Analysis Of Sign Changes:**

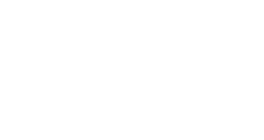
The first column has three sign changes (from 1 to 3 to 0 to 4), indicating that there are three poles in the right-half plane.

**Interpretation:**Therefore, the system represented by q(s) = s^8 + 3s^7 + 5s^6 + 6s^5 + s^4 + 4s^3 + 4s^2 + 2s + 1 is **unstable**.

**Question 2 :**



Y(s)



1

𝑆

(

𝑆

+

1

)

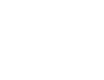
(

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+

5

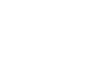
)



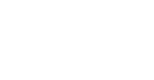
K



+



\_



R(s)

The feedback system consists of a block with transfer function G(s)= s(s+1)(s+5)K​ and a unity feedback path (H(s) = 1).

**Closed-Loop Transfer Function:**

The closed-loop transfer function (W(s)) of the system can be derived using the following formula:

W(s) = 1 + G(s) / H(s)G(s)​

In this case:

W(s) = s(s+1)(s+5)+K / K

**Stability Analysis:**

The stability of the closed-loop system is determined by the roots of the characteristic equation, which is obtained by setting the denominator of the closed-loop transfer function to zero:

s(s+1)(s+5) + K = 0

This expands to a third-order polynomial:

s^3 + 6s^2 + (5 + K)s + K = 0

For the system to be stable, all the roots of this polynomial must lie in the left-half of the complex plane (i.e have negative real parts).

**Applying Routh-Hurwitz Criterion:**

For the given system, the characteristic equation is:

s^3 + 6s^2 + (5 + K)s + K = 0

**Conclusion:**

This implies that the gain K can be any positive value. The system remains stable for any positive value of K.

q(s) =k + s^3 + 6s^2 + 5s = 0

|  |  |  |  |
| --- | --- | --- | --- |
| S^3 | 1 | 5 | 0 |
| S^2 | 6 | k | 0 |
| S^1 | (30 – k)/ 6 | 0 | 0 |
| S^0 | k |  |  |

**For stability , 0 < k < 30**

**Question 3 :**

𝐾

𝐿(s) =

s(s + 1)(0.1s + 1)

1. **For the Nyquist Plot:**

𝐾

𝐿(s) =

s(s + 1)(0.1s + 1)

𝐾

* 𝐿(ω) =

ω (ω + 1)(0.1 ω+ 1)

𝐾

* 𝐿(jω) =

jω (jω + 1)(0.1j ω + 1)

𝐾

* /𝐿(jω)/ =

ω (ω^2 + 1)(0.01ω^2 + 1)

* Θ(ω) = - π/2 - tan⁻¹( ω) - tan⁻¹(0.1 ω)

For ω = 0; /𝐿(jω)/= **∞;** Θ(ω) = - π/2

For ω = **∞;** /𝐿(jω)/= **0;** Θ(ω) = - 3π/2

Matlab Code:

% System parameters (marginal stability case)

k = 1;

num = [k];

den = [0.1 1.1 1 0];

sys = tf(num, den);

% Plot Nyquist diagram (approximate)

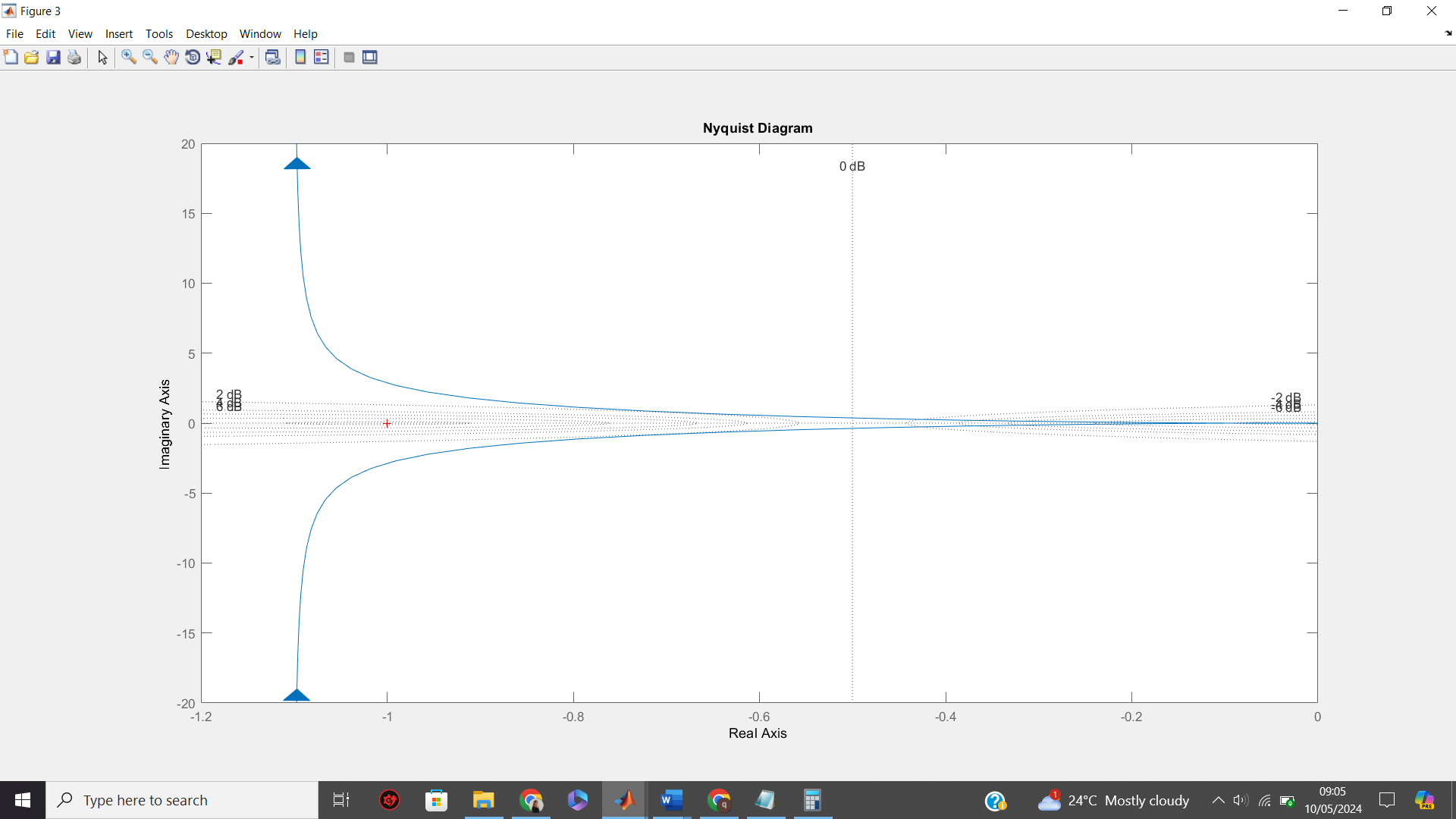
figure;

nyquist(sys);

hold on; % Plot imaginary part vs magnitude for approximation

title('Nyquist Diagram');

grid on;



1. **The maximum value of K for which the system is marginally stable**

The polar diagram crosses the negative real axis at (-1+0j) , when Θ(ω) = -π, if the system is marginally stable.

We have that: Θ(ω) = - π/2 - tan⁻¹( ω) - tan⁻¹(0.1 /10) = - π

* tan⁻¹(11wc /10.wc^2) = π/2



From above; 10 – wc^2 = 0 => wc = 10



At (-1,0j) => **k = 11**

**c) For the Bode Plot:**

For k = 10,

𝐾

𝐿(s) =

s(s + 1)(0.1s + 1)

10

* /𝐿(jω)/ =

ω (ω^2 + 1)(0.01ω^2 + 1)



GdB = 20log10 – 20log w - 20log 1 + w^2 – 20 log 1 + 0.001w^2



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **w** | **0.01** | **0.1** | **1** | **10** |
| **GdB** | 60 | 39.96 | 16.95 | -23.05 |

**Gain Margin** = 1 / /L(jw)/ = 11 / 10 = 1.1

**Phase Margin** = 180 ° + (- 10 - tan⁻¹( 0.79) - tan⁻¹(0.79 /10))

= 47.17 °

This signifies the relative stability of the system.

**MatLab Code:**

Compute Bode response



[mag, phase] = bode(num, den, w);

% Plot Bode diagram

figure;

semilogx(w, mag, 'b');

hold on;

semilogx(w, phase, 'r');

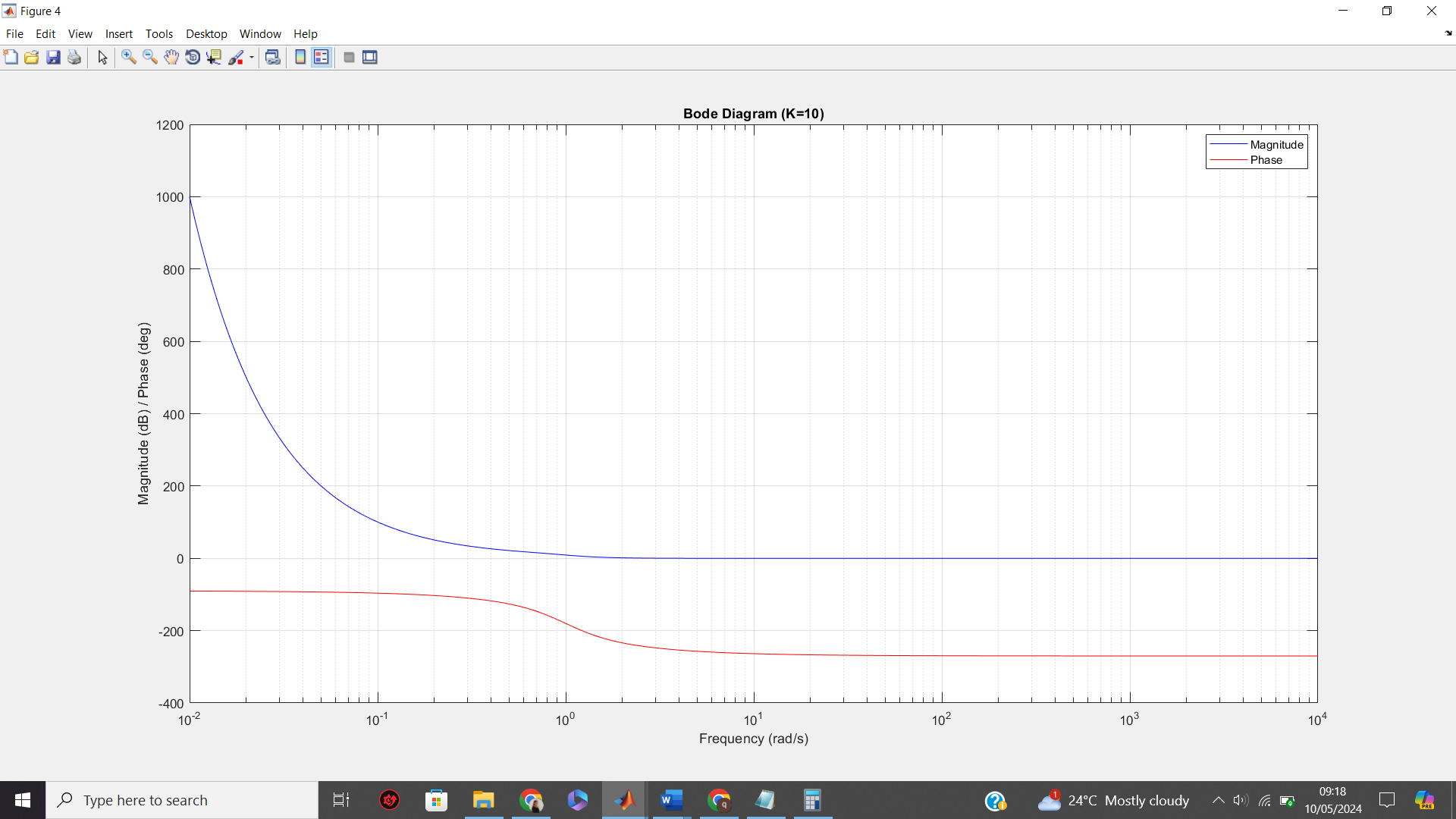
title('Bode Diagram (K=10)');

xlabel('Frequency (rad/s)');

ylabel('Magnitude (dB) / Phase (deg)');

legend('Magnitude', 'Phase');

grid on;



**Question 4 :**

𝐾

𝐿(𝑆) =

𝑆(𝑆 + 1)(𝑆 + 2)(𝑆 + 4)

1. Number of Loci : m = 0, n = 4

Origin of Loci : s = 0, -1, -2, -4

Centre of asymptotes =( poles - zeroes)/ n-m

= - 7 / 4 = -1.75

* Angle of asymptotes : Θ(ω) = (180 + 360j) / 3

Θ = { 60, 180, 300}

**Matlab Code:**

% System parameters

K = 1; % Gain (adjust as needed)

num = K;

den = [1 1 3 4]; % Denominator with coefficients

% Create the root locus object

sys = tf(num, den);

rlocus(sys);

% Title and labels

title('Root Locus Diagram');

xlabel('Real');

ylabel('Imaginary');

grid on;

% Breakaway points calculation

% Use rlocus function with additional argument 'Breakaway'

[r, k\_breakaway] = rlocus(sys, 'Breakaway');

% Display breakaway points (if any)

if ~isempty(k\_breakaway)

hold on;

scatter(real(r), imag(r), 'o', 'MarkerSize', 10, 'MarkerEdgeColor', 'r');

for i = 1:length(k\_breakaway)

text(real(r(i)), imag(r(i)), strcat('K = ', num2str(k\_breakaway(i))), ...

'HorizontalAlignment', 'center', 'VerticalAlignment', 'middle');

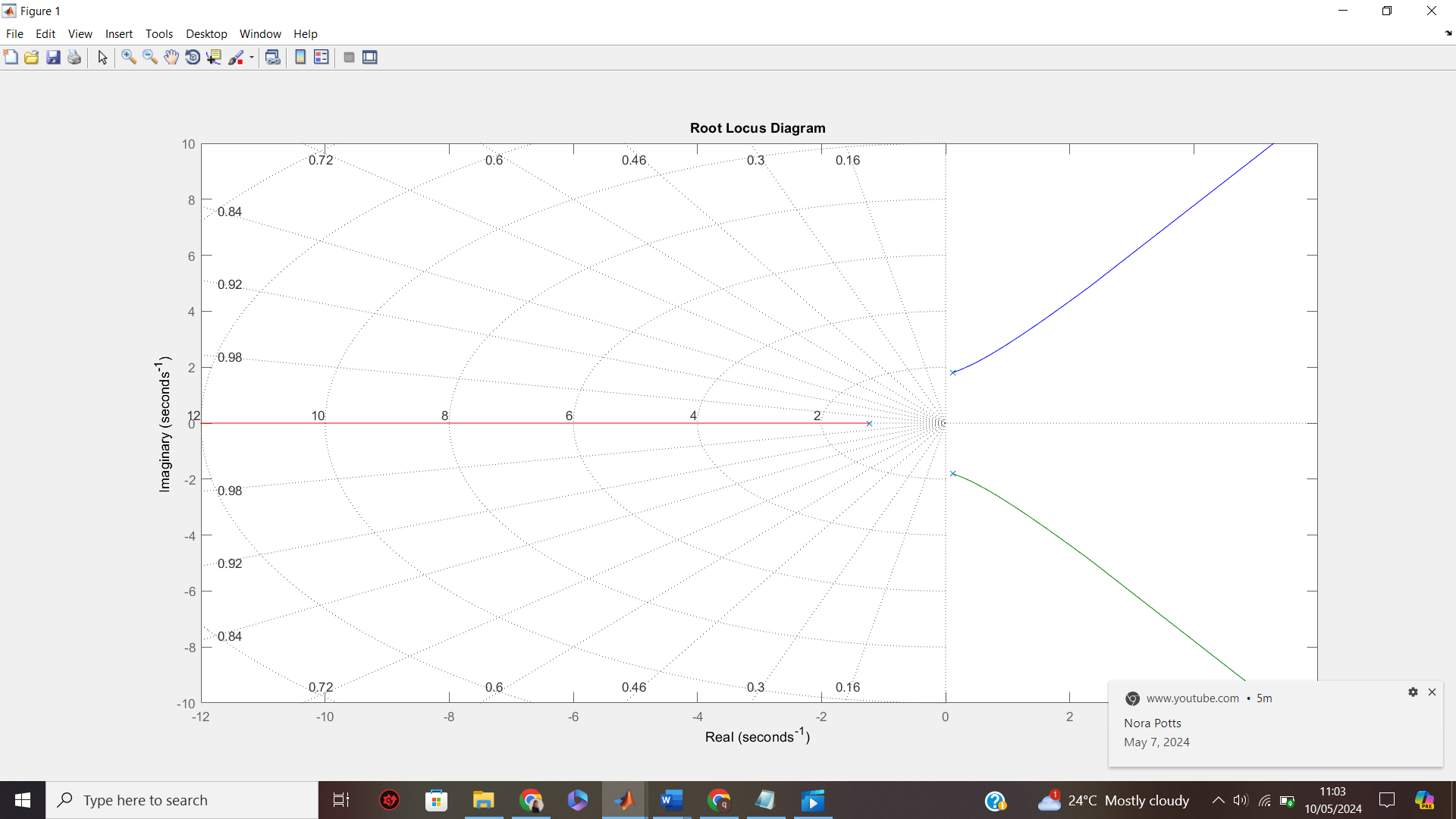
end

legend('Root Locus', 'Breakaway Points');

else

disp('No breakaway points found.');

end



1. q(s) = k + s(s+1)(s+2)(s+4) = 0

= k + 2s^3 + 20s^2 +8 = 0

* k = - 2s^3 - 20s^2 – 8
* dk/ds = -6s^2 – 40s – 8 = 0, Using the quadratic formular s = - 0.21 or s = -6.460

Therefore the break-away point is **s = - 0.21**

1. 𝐾

𝐿(𝑆) =

𝑆(𝑆 + 1)(𝑆 + 2)(𝑆 + 4)

k +s^3+6s^2+s^3+6s^2+8s = 0

* 2s^3 +20s^2+8s+k = 0

|  |  |  |  |
| --- | --- | --- | --- |
| S^3 | 2 | 8 | 0 |
| S^2 | 20 | k | 0 |
| S^1 | (160 – 2k)/ 20 | 0 | 0 |
| S^0 | k |  |  |

For stability, k > 0 => 160 > 2k

Therefore **0 < k < 80**

**Question 5 :**

𝐾(𝑆 + 0.5)(𝑆 + 1.5)

𝐿(𝑆) =

𝑆(𝑆 + 1)(𝑆 + 2)(𝑆 + 4)

1. L(s)= K(s+0.5)(s+1.5) / s(s+1)(s+2)(s+4)

Number of loci ;m=2, n=4

Origin of loci: s=0,-1,-2,-4

Destination of loci: s=-0.5,-1.5

Angle of asymptote=( poles - zeroes)/ n-m

= (-7+7)/2 = -5/2=-2.5

Θ(ω) = (180+360j)/2, j=0,…,2

Θ(ω) = { 90, 270}

**Matlab Code:**

% System parameters

K = 0.1; % Initial gain value (adjust as needed)

num = K \* [1 0.5 1.5]; % Numerator with gain and zeros

den = [1 1 3 4]; % Denominator with coefficients

% Create the root locus object

sys = tf(num, den);

% a) Root Locus diagram and Stability Analysis

figure(1);

rlocus(sys);

title('Root Locus Diagram');

xlabel('Real');

ylabel('Imaginary');

grid on;

% Analyze stability from the root locus plot

all\_stable = all(real(roots(sys)) < 0);

if all\_stable

disp('System is stable for all positive K values.');

else

disp('System is not stable for all positive K values.');

end

% b) Breakaway Points calculation

[r, k\_breakaway] = rlocus(sys, 'Breakaway');

% Display breakaway points (if any)

if ~isempty(k\_breakaway)

hold on;

scatter(real(r), imag(r), 'o', 'MarkerSize', 10, 'MarkerEdgeColor', 'r');

for i = 1:length(k\_breakaway)

text(real(r(i)), imag(r(i)), strcat('K = ', num2str(k\_breakaway(i))), ...

'HorizontalAlignment', 'center', 'VerticalAlignment', 'middle');

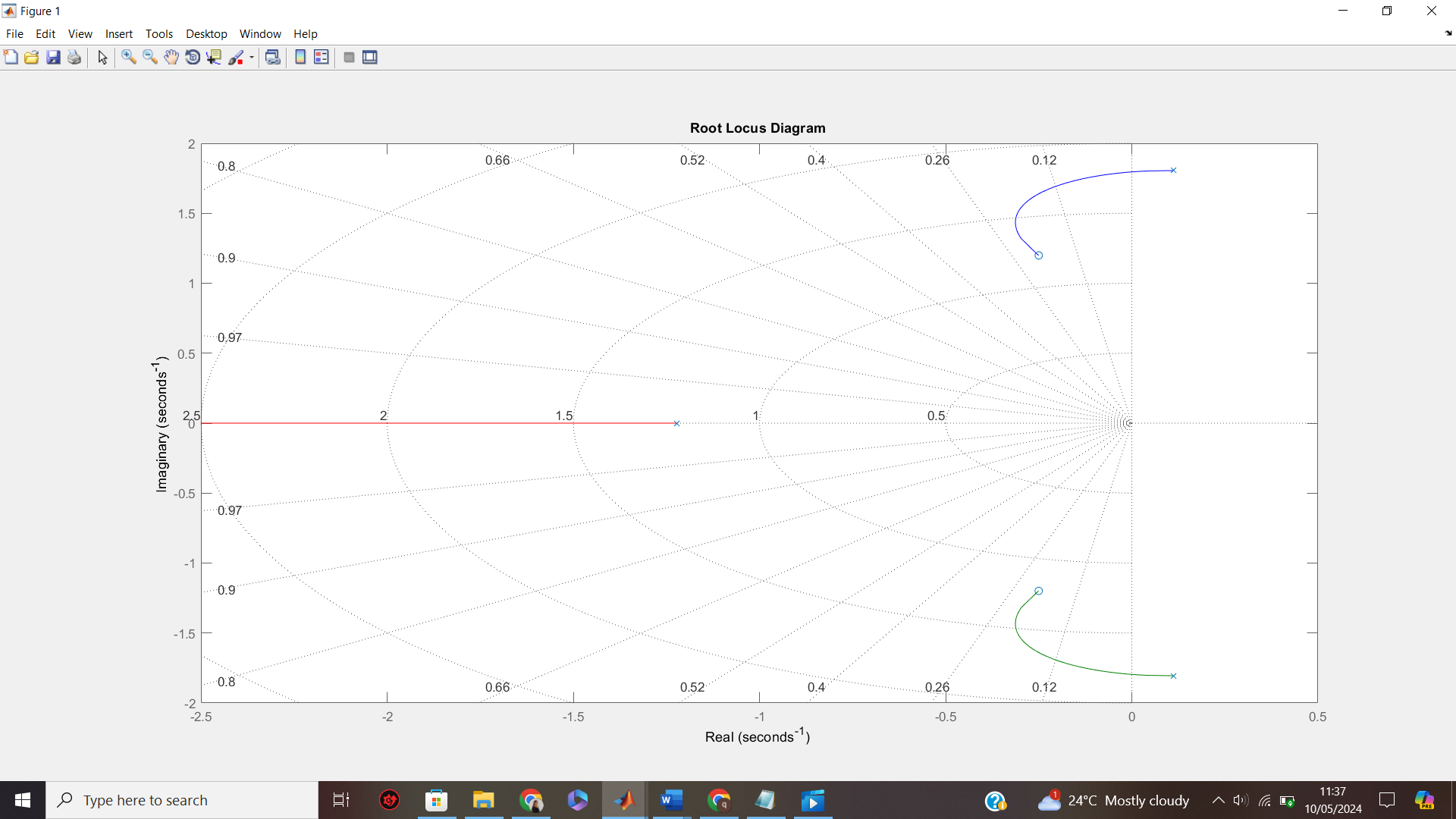
end

legend('Root Locus', 'Breakaway Points');

else

disp('No breakaway points found.');

end



1. k(s+0.5)(s+1.5)+2s3+20s2+8s=0

k(s+2s+0.75)+2 s3+20 s2+8s

2 s3+ (20+k) s2+(8+2k)s+0-75=0

K=(2 s3+20 s2+8s)/ s2+2s+0.75

dk/ds=( s2+2s+0.75)(6 s2+40s+8)-25

=[( s2+2s+0.75)(6 s2+40s+8)-(2 s3+20 s2+8s)(2s+2)]/( s2+2s+0.75) 2

S= -2.2778

S2=-0.5966

S3=0.6601+1.1632

S4=0.6601-1.1632

Break away, s2

2s+(20+k)s+(8+2k)s+0.75=0

|  |  |  |
| --- | --- | --- |
| S3 | 2 | 8+2k |
| S2 | 20+k | 0.75 |
| S1 | [(20+k)(8+2k)]/20+k | 0 |
| S0 | 0.75 |  |

For stability,

160+40k+2k2-1.5

K=18.43 or k= -22.43

K<-22.43 or k>18.43.

**Question 6 :**

****

L(s)= ks(s+2) / (s+2)(s+1)

Num of loci, m=2, n=2

Origin of loci s=0,-1,-2,

Center of asymptode ∑poles-∑zeros/n-m

-7-0/4

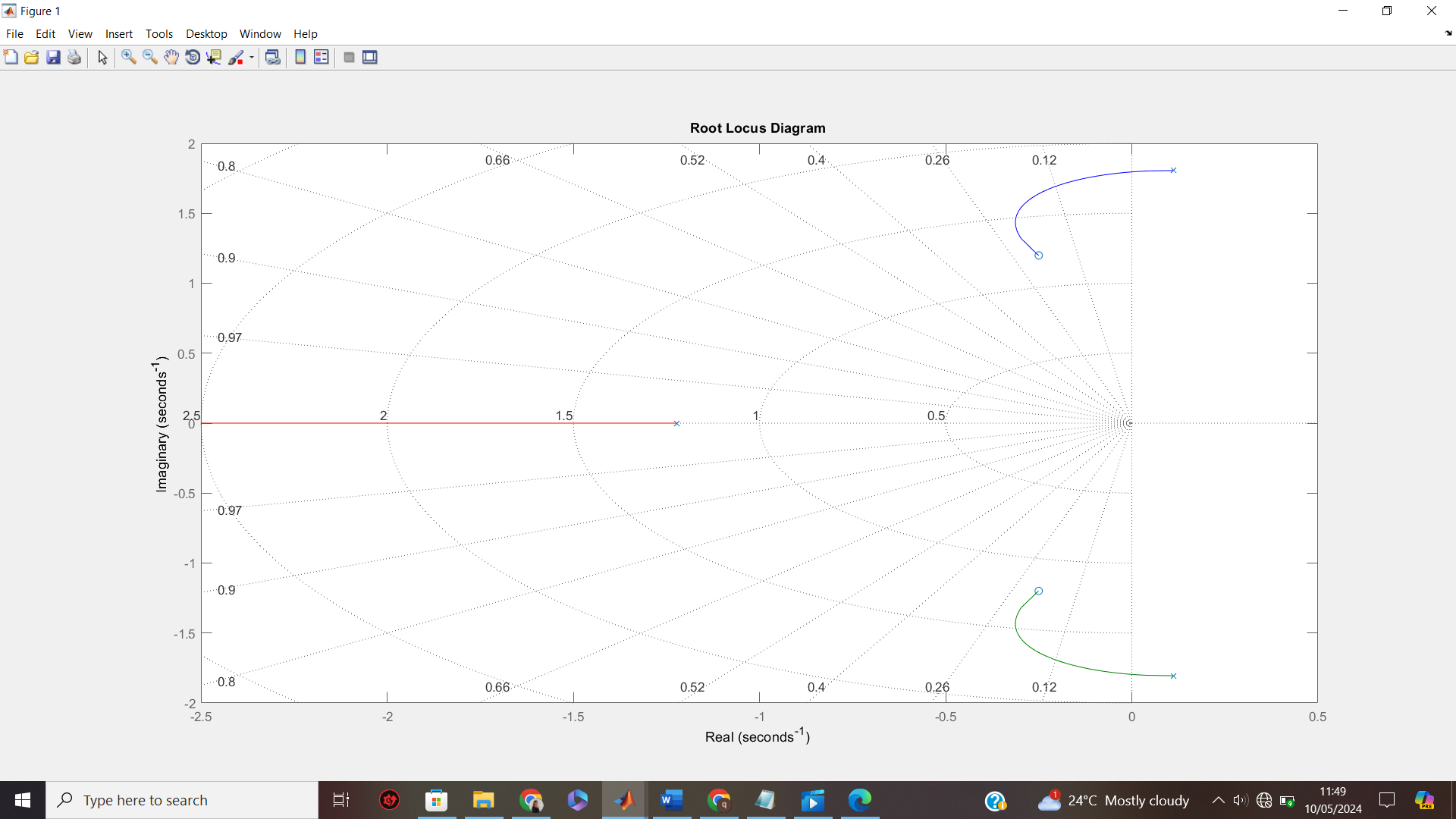
-7/4= -1.75

Angle of asymptode

Φ=180+360j, J= 0,……4

Φ=60, 180, 300

Diagram.



b)q(1)=k+s(s+1)(s+2)(s+4)=0

k+(s+s2)(s+6s2+8)=0

k+s3+6s2+8s2+s+6s2+8s=0

k=-2s3-20s2-8s=0

dk/ds=, 3s3+20s2+4=0

s

s=-0.21 or -6.460

break away, s= -0.21

c)k/s(s+1)(s+2)(s+4)

k+(s2+s)(s2+6s+8)=0

k+s3+6s2+8s2+s3+6s2+8s=0

2s3+20s2+8s+k=0

|  |  |  |  |
| --- | --- | --- | --- |
| s3 | 2 | 8 | 0 |
| S2 | 20 | k | 0 |
| S1 | (160-2k)/20 | 0 |  |
| S1 | k |  |  |

For stability, k>0

(160-2k)/20>0

Therefore 160>2k

K<80

0<k<80